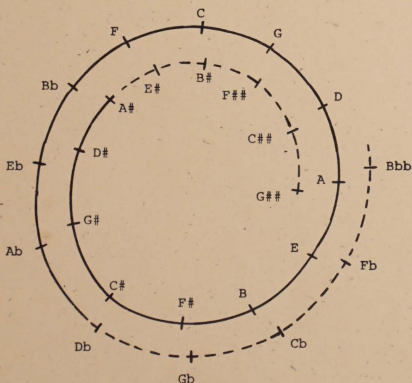


PHYSICS. MUSIC THEORY and the HAMMERED DULCIMER

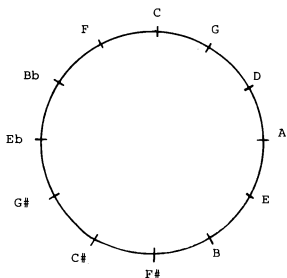
by RICK FOGEL



Endless series of perfect fifths.
(G##, A & Bbb are different pitches)

Also by Rick Fogel with N.A. Martin

Building a Hammered Dulcimer, Do-Kit-Yourself



Circle of equal fifths.

copyright © 1979 by Rick Fogel

First Printing, January 1979
Second Printing, August 1981
Third Printinnng, August 1996
Fourth Printing, February 2003

PREFACE

The purpose of this work is to concisely connect physics, music theory and the hammered dulcimer, and I hope to interest the amateur, as well as the serious student, of string music. The hammered dulcimer (and in some cases the violin) is used for examples because it is easily tuned and played in different intonations. I have assumed the reader to be unfamiliar with science or music.

The tonic (or key note) used for examples has been C (0 cents). The notation used has been that of natural intonation because this intonation is so basic to an understanding of music theory, and the natural scale has artistic value. For a thorough understanding of the natural scale it is important to be able to hear the exact intervals and pitches spoken of throughout this work, and as it is quite impossible to do so in any ordinary instrument, I have contrived a specially-tuned hammered dulcimer described in Appendix I. Appendix II on the wind-blown or Aeolian dulcimer has been added for fun. Appendix III defines the unit 'cents' used for musical intervals. The leaflet entitled *The Hammered Dulcimer* has been added to describe playing, building and buying a hammered dulcimer.

I am indebted to friends as well as books. A considerable fraction of this work is merely *Sensations of Tone* by Hermann Helmholtz shortened and simplified. Also included is information by A. J. Ellis, the English translator of Helmholtz's book. I received many helpful comments on the manuscript from Howie Mitchell, Pete Vigour and Arlene Fogel. Kathie Dunning helped me with most of the typing. Other sources which were especially valuable are:

Science and Music by Sir James Jeans (Cambridge University Press)
The Physics of Music by Alexander Wood (Chapman and Hall)
Making a Hammered Dulcimer and Hammer Dulcimer History and Playing by Sam Rizzetta (Smithsonian Institution)
The Hammered Dulcimer Instruction Book by Phillip Mason (Communications Press, Inc.)
The Hammered Dulcimer by H. Mitchell (Folk-Legacy Record, Inc.)

On the personal side, I am especially indebted to my father and to Martha I. Covington.

RICK FOGEL

January, 1979

CONTENTS

PREFACE

PART I

Musical Tones	1
Consonant Intervals	1
Ohm's Law	3
Musical Tone of Struck Strings	4
Sympathetic Resonance	4
Musical Tone of the Hammered Dulcimer	5
General Rules for Quality of Tone	6
Time of Reverberation	7
Combinational Tones	9
Beats	9
Consonance and Dissonance	11
Harmoniousness of Chords	14
Summary of PART I	16

PART II

Scales	17
Harmony of the Modes	20
Discords	22
Keys and Modulation	23
Comparison of Intonations	23
Aesthetical Relations	26

APPENDICES

I Natural Intonation for the Hammered Dulcimer	28
II Aeolian Dulcimer	29
III Cents	29

SUPPLEMENT

Rick Fogel Hammered Dulcimers

INDEX	30
-------	----

GLOSSARY	31
----------	----

ERRATA	31
--------	----

PART I

MUSICAL TONES

The whole sensation excited in the ear by a periodic vibration of the air we call a musical tone. Musical tones of strings are distinguished by the following: (1) force, (2) frequency and (3) quality.

(1) The force of a musical tone from a struck string is greatest at first, sometimes the vibrations are visible, then these vibrations become smaller and at the same time the loudness diminishes and the tone has less force (the amplitude decreases with time).

(2) The frequency is the number of vibrations per second of the string (also called pitch).

(3) By quality of tone we mean that peculiarity which distinguishes the tone of a violin from a hammered dulcimer or other instrument when each produces a note of the same frequency. The reason for the different qualities of tone is the different force of each partial tone in the compound tone of each instrument.

In most cases, musical tones are compound tones, containing a series of different simple tones called partial tones. The first partial tone is the prime, the rest are its upper partial tones. The number shows the order of a partial tone, and shows how many times its frequency exceeds the frequency of the prime. Therefore, the second partial tone makes twice as many, the third, three times as many vibrations in the same time as the prime, and so on. See Figure 1.

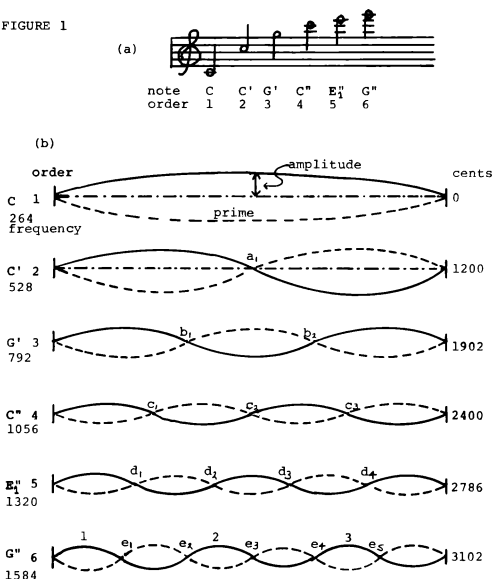
CONSONANT INTERVALS

A musical tone is said to be an octave ($1/2$) higher than another when it makes exactly twice as many vibrations in a given time as the latter. Therefore, the second order partial forms an octave with the first or prime, and the fourth partial forms an octave with the second partial and a double octave ($1/4$) with the prime. Two musical tones stand in the relation of a fifth ($2/3$) when the higher tone makes three vibrations in the same time as the lower makes two. Therefore, the third partial makes a fifth with the second partial and a twelfth ($1/3$) with the prime. The sixth partial makes a fifth with the fourth partial. Similarly the fifth partial tone makes a third ($4/5$) with the fourth partial tone.

The major chord of three tones consists of a major third and a fifth with ratios as follows (subscripts and superscripts are defined on Page 18):

$$\begin{array}{l} C : E_1 : G \\ 1 : 4/5 : 2/3 \end{array}$$

FIGURE 1



The solid line represents the string at the beginning and end of one vibration. The dash line represents the string halfway through one vibration. The dash-dot line is the equilibrium position. Compound tones of good musical quality contain all 6 of these partials.

(c) transverse vibrations of strings

$$f = 1/2L \sqrt{P/m}$$

f = frequency in cycles/second (Hz)
 L = length of string in meters
 P = tension of string in Newtons
 m = mass per unit length in kg/meter

Thus we see that this major chord is composed of tones which are one and/or two octaves lower than partials of the tonic or prime tone C. Associated with the tonic chord is that of its dominant G : B₁ : D and that of its sub-dominant F : A₁ : C, each of which has one tone in common with the tonic chord. Combining these chords we obtain the complete series of tones for the major scale of C:

$$\begin{array}{cccccccc} \text{C} & : & \text{D} & : & \text{E}_1 & : & \text{F} & : & \text{G} & : & \text{A}_1 & : & \text{B}_1 & : & \text{C}' \\ 1 & : & 8/9 & : & 4/5 & : & 3/4 & : & 2/3 & : & 3/5 & : & 8/15 & : & 1/2 \end{array}$$

Thus we see that the tones of the major scale are also lower octaves of the first five partial tones of the tonic, dominant and sub-dominant. This scale is called the natural or just scale and will be discussed further in Part II.

OHM'S LAW

A law by G. S. Ohm states that a composite mass of musical tones (from one or more sounding bodies) is capable of being analyzed into a sum of a number of simple tones, sensible to the ear, corresponding to the partials of each musical tone. One method of observing the partial tones of a compound musical tone is as follows: STEP 1 - On any string instrument, produce the tone we wish to hear as a harmonic by touching the corresponding node of the string when it is struck. STEP 2 - Strike the string without touching the node. Thus we learn to recognize the existence of the tone of Step 1 as part of the tone of Step 2. In other words, to hear the third order partial, divide the string visually into three equal parts, experimentally find the node (b₁ or b₂ in Figure 1) by touching the string gently with the finger in the neighborhood of the node, then striking the string and moving the finger about until the required harmonic sounds strong and pure. Next, strike the string without touching the node. Thus we hear the third partial twice, first as a harmonic and second as part of the compound tone of the whole string. This procedure may be carried out until the upper partials are too weak to hear. The loudness of the partial tones diminishes as their ordinal number increases. For struck strings, the intensity of the partial of order *m* is proportional to 1/*m*, and for plucked strings, 1/*m*². The upper partial tones are therefore louder for struck rather than plucked strings.

On the other hand, if the frequencies of the upper partials are ever so slightly different from their stated value, then the tone ceases to make that uniform impression corresponding to the harmonic upper partials just discussed. To hear this effect, fasten a small piece of wax on any part of a musical instrument's string. Even a very small piece of wax does considerable harm to the tunefulness of the sound. The tone becomes dull and rough corresponding to inharmonic upper partials (upper partials which are slightly different from those given in Figure 1, Page 2). This effect is also observed when strings become old or dirty. There is further discussion on Page 9.

MUSICAL TONES OF STRUCK STRINGS

The intensity (or force) of the prime tone and upper partial tones in a struck string depends in general on: (1) the nature of the stroke, (2) the place struck and (3) the thickness and material of the strings.

(1) On the nature of the stroke: The string may be struck with a sharp-edged metallic hammer which rebounds instantly. This produces the most abrupt discontinuities in the string and a corresponding long series of upper partial tones. The tone will be of poor quality and sound tingling and piercing. When the hammer is soft and elastic, the motion of the string spreads before the hammer rebounds. The discontinuity of motion is much less (diminishing as the softness of the hammer increases) and the intensity of the higher upper partial tones is decreased. The tone is thus softer and more harmonious and the prime tone is louder.

(2) On the place struck: Those partials are comparatively strongest which have a maximum displacement at the point struck. Conversely, those partials disappear which have a node at the point struck. Therefore, the composition of the musical tone of strings can be greatly varied by changing the striking point. For a string struck in its middle, the even-numbered partials will disappear and the sound will be hollow or nasal. For a string struck at $1/3$ its length, the third, sixth, ninth and etc. partials disappear and the sound will be somewhat hollow but less than when struck at the middle. When the point struck approaches the end of the string, the prominence of the higher upper partials is favored at the expense of the prime (and lower upper partials) and the sound becomes poor and tinkling.

As we have already seen, the first six partials of a compound tone contain only octaves, fifths and the major third of the prime tone. The seventh and ninth partial tones do not belong to the major chord (being B $\flat$ and D'' when C is the prime). Therefore, a good striking place for a string is from $1/7$ to $1/9$ of its length.

(3) On the thickness and material: The more rigid and/or thicker the material, the less the higher upper partials will sound because the string cannot readily vibrate with short sections. The thinner and/or more flexible strings are better for producing the higher upper partials. The heavier and/or thicker strings will give a greater total intensity of sound.

SYMPATHETIC RESONANCE OF STRINGS

Sympathetic resonance is when a string is excited by another string in its neighborhood. This happens when the frequency of their primes and/or one of their upper partials are in common.

To make experiments on the hammered dulcimer, put a small piece of folded paper on the C' string. The paper will be put into motion (even thrown off) when C or another C' is struck. Less motion of the paper will be seen if one

of the partials of C' is struck (C" or G"). Thus we have another way in which to verify the existence of the upper partials.

When the frequency of the string which is meant to vibrate in sympathetic resonance is not exactly the same as the sympathizing string, the former will nevertheless often make sensible sympathetic vibrations, which will diminish in intensity as the difference of frequency increases.

MUSICAL TONE OF THE HAMMERED DULCIMER

There is an interesting addition to the compound tone of a note struck on the hammered dulcimer due to the fact that there are two tones per string, one on either side of the bridge. The reason for this addition is that the bridge does not completely reflect the waves of the string as the fixed end of the string does. Therefore, energy is transferred across the bridge to weakly excite the tone of the other portion of the string. There will be less energy transfer across a very firm bridge or when more of the string is in contact with the bridge cap. Thus a wide bridge cap or a sharp angle of the string at the bridge will lessen this effect.

For example, place the bridge at $\frac{2}{3}$ the length of the string so that the tone on the left side is a fifth interval above the tone on the right. When striking the note on the right, the tone on the left, a fifth higher, also sounds weakly, which is an octave lower than the third order partial of the note struck. When striking the tone on the left, the tone on the right is an octave lower than the fourth of the note struck. Therefore, in both cases the extra tone is not among the lower upper partials of the note struck, although it is consonant with it. This effect gives a unique compound tone to the hammered dulcimer. Similar effects are observed when the bridge is placed at other intervals (as $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$).

So far we have referred to vibrations of the strings alone. Vibrating strings do not directly communicate any sensible portion of their motion to the air, but the motion of the string is communicated to the air by the vibrating body of an instrument. Each partial tone of the string, however, is not equally well communicated to the air by the instrument with precisely the same degree of intensity as the intensity they possess on the string itself. The strings of the hammered dulcimer agitate the bridge over which they are stretched. The bridge stands near a firm support which acts as a pivot for the top of the instrument, and the support also transmits vibrations to the back. The exact distance between this internal brace and the bridge is best determined by listening to the quality of the tone of the individual instrument.

An appropriate structure of the instrument and wood of the most perfect elasticity are probably the most important conditions for regular vibrations of the strings. This allows for a pure flow of tone from the strings to the instrument to the air without any roughness. Good

instruments consequently allow for a much more powerful motion of the strings and the whole intensity of their tone can be communicated to the air without diminution, whereas the friction caused by any imperfection in the elasticity of the wood destroys part of the motion. Much of the advantages of old instruments may, however, depend upon their age and especially their long use, both of which act favorably on the elasticity of the wood.

An enclosed mass of air like that inside the hammered dulcimer has certain proper tones or resonances which may possibly be evoked by blowing across the openings. Another method is to place the ear against the back of the instrument and play a scale on a piano. Some tones will be found to penetrate the ear with more force than others owing to the resonance of the instrument. The consequence of this resonance is that those tones of strings which lie near the proper tones of the instrument are especially prominent.

The musical quality of tone of the hammered dulcimer is durable and full because the metal strings are under much tension, are attached to a strong frame and pass over heavy bridges that cannot be shaken very much. Therefore, they give out their vibrations slowly to the air and sounding board but their vibrations continue for a relatively long time. In comparison, instruments such as the violin or guitar with strings of less mass (sometimes comparable mass), fastened with light supports, passing over light bridges supported by a very mobile sounding board, and for the violin the part the strings touch is slightly elastic as when the soft finger stops a string, their vibrations rapidly disappear and the tone is short and without ring, but more powerful and penetrating.

On many hammered dulcimers there is no damping mechanism to completely stop the continuance of sound after each note is struck. Therefore, the sound may be discordant in certain combinations of notes (consecutive tones in a scale for example). This confusion can often be removed by playing in the higher octaves which are usually relatively poor in upper partials (the deeper octaves are usually rich) or by slowing down the tempo to allow the individual tones more time to diminish in intensity or by simply playing softer. For other combinations of notes, the continuance of sound may be concordant (as with tones of a chord).

GENERAL RULES FOR QUALITY OF TONE

(1) Simple tones, like those of tuning forks (where only the prime is heard since they have no upper partial tones) have a very soft quality but lack power. The tones of flutes approach simple tones.

(2) Musical tones where the first six partials are audible with the seventh and higher inaudible are considered the most harmonious and musical. The tones produced by the piano, open organ pipes and the softer tones of the human voice are examples.

(3) When only the uneven numbered partials are present (as in strings struck in their middle and clarinets), the

tone is hollow, and when a large number of upper partials are heard, nasal.

(4) When the prime tone predominates, the quality is rich, and when the prime is not sufficiently stronger than the upper partials, the quality of tone is poor. Strings struck with piano hammers give tones of a richer quality than when struck by a hard hammer or plucked by the finger.

(5) When partial tones higher than six are very distinct, the tone is cutting and penetrating and sometimes rough (as in the brass instruments). The reason is the dissonances the upper partials form with one another. When the force is weak, the higher upper partials are useful in giving character and expression to the music (for example, the violin and human voice).

TIME OF REVERBERATION

The fundamental fact to be understood is that the room in which music is performed is an extension of the instrument or voice; the tone is modified by the characteristics of the room. The most important characteristic is the time of reverberation; in other words, the time which sound takes to die away after the source has ceased to operate.

$$T = 0.16 V / A$$

T is time of reverberation in seconds.

V is volume of room in cubic meters.

A is total absorption in open window units.

The absorption of sound-energy is proportional to the intensity of sound-energy. Thus, for a constant rate of sound-energy production in a room, the rate of absorption is at first small, but gets greater as the intensity gets greater. Finally, a balance is achieved where the rate of energy which is put in is equal to the rate of energy absorbed, and the intensity remains constant. When the source ceases to sound, absorption continues at a gradually diminishing rate. The amount of absorption in a room determines: (1) the time to reach a steady intensity level, (2) the level of intensity reached, (3) the time for intensity to fall to zero after the source ceases. All the sound which falls on the space of an open window passes out of the room, and therefore, so far as the room is concerned, is completely absorbed. Thus, an open window unit is the absorption of one square meter of open window. The total absorption of a room is

$$A = c_1a_1 + c_2a_2 + c_3a_3 + \dots$$

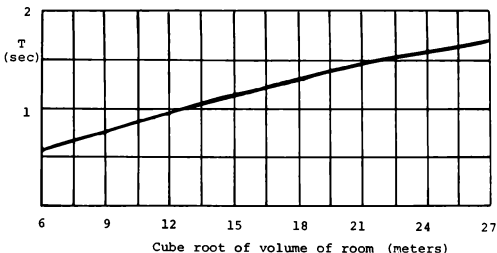
where the c's are the coefficients of absorption (given in table below) and the a's are the areas of the corresponding material. Note that the coefficients of absorption vary with frequency. As a rule, they increase with increasing frequency.

If T is long for low-pitched tones and short for high-pitched tones then all music will be more mellow or even dull (blurred if too long). If it is short for low-pitched tones and long for high-pitched tones, then the quality is more brilliant or possibly thin (dead if too short). Generally, performers like more reverberation than the audience, therefore, keep hard reflecting surfaces near performers. See graph for ideal values of T.

Table of Absorption Coefficients c

<u>Material</u>	<u>Frequencies in cycles/sec</u>		
	<u>250</u>	<u>500</u>	<u>1000-2000</u>
Plaster	0.02	0.03	0.03
Brick	0.02	0.02	0.04
Wood paneling	.01-.02	.01-.02	.01-.02
Curtains (light to heavy)	0.2-1.0	0.2-1.0	0.2-1.0
Audience (per person)	0.43	0.47	0.50
Wood Floor	0.03	0.06	0.10
Carpet	0.05	0.20	0.50
Furniture	0.1-1.0	0.1-1.0	0.1-1.0

Ideal Values of T



Reverberation time (T) for rooms acknowledged good for musical tone and speech. A longer T seems more suited for orchestral music, and a still longer T, for choral music.

COMBINATIONAL TONES

A composite mass of musical tone reaches the ear as the sum of the individual simple tones plus sensations due to other phenomena. One phenomena is combinational tones. These are heard whenever two simple tones of different frequencies sound simultaneously, loudly and continuously. The frequencies of the combinational tones are equal to the difference and summation of the frequencies of the generating simple tones. The difference tones are louder and stronger than the summation tones. On the hammered dulcimer, combinational tones are not strong enough to be heard except at the first instant the strings are struck, and then they die off sooner than the generating tones. Audible combinational tones are not particularly sensitive to different intonations or to tuning within that intonation.

On the violin, sound the open A and E strings loudly and continuously. The difference tone a on the g string will be clearly audible. Some violinists listen for this lower octave differential tone when tuning their perfect fifths. Also on two flageolets blown strongly at different high tones as G'' and F#'', one hears a note near g below middle C - the difference tone.

In treating the relative harmoniousness of intervals, combinational tones may produce beats (to be considered next). The combinational tones from any single compound tone coincide exactly with the partials themselves and give no beats. Thus if the prime makes n vibrations/sec, the upper partials make $2n$, $3n$, $4n$, etc. vibrations/sec, and the differences of these numbers are again n , $2n$, $3n$, etc. If, on the other hand, the upper partials are slightly different from these, the combinational tones will differ from one another and from the upper partials and the result will be beats. This is the reason for the effect of the inharmonic partials of a string with a small piece of wax attached discussed on page 3.

Harmonic tones are generated when vibrating bodies undergo asymmetrical distortion. For example, on one side of the ear-drum is air; on the other side is a bony structure. When the ear-drum moves in an outward direction, only its own elasticity pulls it back; when it moves inward, its motion is further impeded by this bony structure. For a tone of frequency f this asymmetry in the ear produces the frequencies $2f$, $3f$, $4f$, etc. - the full harmonic series. Similar phenomena happen in other parts of the ear and in musical instruments. The effect of double forcing of an asymmetric system is to produce combinational tones.

BEATS

A second phenomenon accompanying the simultaneous sounding of two simple tones is beats. Beats are distinguished from combinational tones as follows: in combinational tones the vibrating bodies undergo certain disturbances which the ear hears as usual; in beats the opposite, the bodies vibrate as usual but the composition of the sensations is disturbed. When two simple tones of

sufficiently different frequency enter the ear together, the sensations due to each are undisturbed because different nerve fibers are excited. But two simple tones of the same or nearly the same frequency which excite the same nerve fibers in the ear produce interference when caused by two perfectly equal simple tones and beats when due to two nearly equal simple tones.

First, interference happens when the vibrations of the two simple tones are different by half the periodic time of the vibrations. Using Figure 1(b), Page 2, in another sense, suppose the solid line to represent the vibration of one source and the dashed line the other source at the same time. When we come to add these two curves, they mutually destroy each other, being always in a contrary direction, giving as their sum a straight line (dash-dot line) or no vibration at all. In another case, when vibrations from two equal sources are always in the same direction, adding them we have a tone not twice, but four times the loudness of either source because the loudness (or intensity) of sound is proportional to the square of the height (or amplitude). These two cases are called destructive and constructive interference.

An example of sound canceling sound is exhibited by every tuning fork, because the prongs vibrate in opposite directions. Strike the tuning fork and slowly revolve it about its long axis close to the ear. You can find four positions in which the tone is heard clearly and four intermediate positions in which it is inaudible. See Figure 2.

Now we consider what happens when the simple tones are of different frequency. The process of beats is shown graphically in Figure 3. Let the marks along the top of the line a-e represent the positive maximum for a frequency of 18 vibrations/sec for 1 second (in Figure 1(b), Page 2, order 6 would have 3 vibrations/sec with the positive maximum marked 1, 2 and 3). Let line f-g be 1 second for a frequency of 20 vibrations/sec. At a, c and e maximums concur and the tone is reinforced. At b and d they are

FIGURE 2

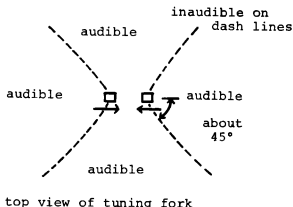
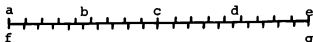


FIGURE 3



intermediate and mutually cancel each other. We thus hear two beats in the second from a to e. Hence, the number of beats is equal to the difference of the two frequencies.

The intensity of tone varies; the maximum intensity times are the beats proper (a, c and e) and these are separated by pauses (b and d). Beats are easy to produce on most instruments by sounding two notes of nearly the same pitch. They are best heard from simple tones such as tuning forks because here the tone completely vanishes in the pauses. For two tuning forks of exactly the same frequency, add a little wax on the end of one to lower its frequency. On the hammered dulcimer, beats are heard whenever the strings of a course are not in unison. This fact can be used when tuning if after one string is properly tuned, strike the course and tune the other string(s) until the beats disappear.

As the interval between two simple tones increases from unison, the beats are at first slow undulations not necessarily unpleasant to the ear. Then, as the interval continues to widen, the beats increase in rapidity and the effect gets rougher until a maximum roughness at about 33 beats/sec, according to Hemholtz, then the roughness is lost as the beats are no longer distinct (greater than 33 beats/sec).

Up to this point we have mostly considered simple tones. For compound tones, beats exist for each combination of prime, upper partials and combination tones. Thus, for each beat of the prime tones, there are two beats between the second partials, three between the third, and so on. The beats of combinational tones occur with all kinds of musical tones, both simple and compound. Of course, the beats of upper partials and combinational tones are found only when such tones are themselves distinct.

CONSONANCE AND DISSONANCE

When two musical tones are sounded at the same time, their united sound may be disturbed by the beats of partials and/or combination tones so that the composite mass of sound is broken up into pulses of tone with a rough effect. This is called dissonance. When there are certain ratios (small whole numbers) between the frequencies of the two tones, there is no unpleasant disturbance of the composite mass of sound and no beats are heard. This is called consonance.

The most perfect consonances are those in which the prime tone of one of the notes coincides with some partial tone of the other. Examples are the octave ($1/2$), twelfth ($1/3$) and double octave ($1/4$). The ratios are of the

frequency of the two prime tones and they also show which partials coincide. For example, for the octave ($1/2$), all partials which form the ratio $1/2$ will coincide, as 1 and 2, 2 and 4, 3 and 6, and so on. See Figure 4.

The next consonances are the fifth ($2/3$) and the fourth ($3/4$), with the fourth being less consonant than the fifth. The intervals are considered harmonious in all parts of the scale. They are the inverse of each other. If the lower tone (the root) of the fifth $C : G$ ($2/3$) is transposed an octave higher to C' (2 becomes 4) we have the fourth $G : C'$ ($3/4$). Or, one could say a fourth is the defect of a fifth from an octave.

The next group consists of the major sixth ($3/5$) and major third ($4/5$). The major third (and the minor third and minor sixth to be considered next) are most harmonious in the higher octaves because the beats between the lower partials, which are a semitone apart, are too rapid to be sensible. Any defect in its intonation produces sensible beats of the upper partials.

The last group consists of the minor third ($5/6$) and minor sixth ($5/8$). These are, in general, not independently characterized, because the partials on which their definition depends are usually very weak (the 6 and 8 order partials). They lie on the boundaries of consonance and dissonance. They are defects of the major sixth and major third from the octave or fifth. Small defects in intonation do not necessarily produce sensible beats.

By increasing the interval by an octave, the harmoniousness of the fifth $C : G$ and the major third $C : E_1$ is improved on, becoming the twelfth $C : G'$ (1902 cents; $(2/3)(1/2)=2/6=1/3$) and major tenth $C : E_1'$ (1586 cents; $(4/5)(1/2)=4/10=2/5$). But, the fourth $C : F$ and major sixth $C : A_1$ become worse as the eleventh $C : F'$ (1698 cents; $(3/4)(1/2)=3/8$) and major thirteenth $C : A'$ (2084 cents; $(3/4)(1/2)=3/8$). The minor third $E_1 : G$ and minor sixth $E_1 : C'$ also become worse as the minor tenth $E_1 : G'$ (1516 cents; $(5/6)(1/2)=5/12$) and minor thirteenth $E_1 : C''$ (2014 cents; $(5/8)(1/2)=5/16$).

The following intervals within one octave are generally considered dissonant: the major and minor seconds and their inverses, the minor and major sevenths, the sharp fourths and the natural seventh $C : ^7Bb$ ($4/7$) (969 cents, where 7Bb is the 7 order partial of C). All of these have ratios with numbers 7 or larger, which are not considered small.

Two C notes which differ from each other by one tenth part of a semitone produce about three beats in two seconds, which cannot be observed without considerable attention. Two C' notes with the same error give three beats in one second and two C'' notes, six beats in one second, which becomes very disagreeable. Hence, high tones are much more sensitive to an error in tuning amounting to a fraction of a semitone, than deep ones.

FIGURE 4

Lower note order	C 1	C' 2	G' 3	C" 4	E" 5 ¹	G" 6
Octave (1/2) order		C' 1		C" 2		G" 3
Fifth (2/3) order	G 1		G' 2		D" 3	G" 4
Fourth (3/4) order	F 1		F' 2	C" 3		F" 4
Major sixth (3/5) order		A ₁ 1	A' ₁ 2 ¹		E" 3 ¹	A" 4 ¹
Major third (4/5) order	E ₁ 1		E' ₁ 2 ¹	B' ₁ 3 ¹	E" 4 ¹	G ₂ [#] 5 ¹
Minor third (5/6) order	E ^b ₁ 1		E ^b ₁ 2	B ^b ₁ 3	E ^b ₁ 4	G" 5

Consonant intervals within one octave (partials 1 to 6 audible)

HARMONIOUSNESS OF CHORDS

As with the intervals of the last section, we are dealing exclusively with the isolated effect of the chord in question, independently of any musical connection, mode, key and modulation. If each one of three tones forms a consonant interval with each one of the other two, the chord is consonant. Combining the consonant intervals (within one octave) found in the last section, we have the results in Figure 5.

The chords of three notes:

major	minor	
(1) C : E ₁ : G 4 : 5 : 6	(2) C : E ¹ _b : G 10 : 12 : 15	Fundamental
(3) C : F : A ₁ 3 : 4 : 5	(4) C : F : A ¹ _b 15 : 20 : 24	Chords of sixth and fourth
(5) C : E ¹ _b : A ¹ _b 5 : 6 : 8	(6) C : E ₁ : A ₁ 12 : 15 : 20	Chords of sixth and third

The fundamental chords have their tones in the closest positions, that is, forming the smallest ratios with each other. The fundamental major chord C+E₁-G is the basis of all other major chords and C-E¹_b+G is the basis for all minor chords. Inverting the intervals within these chords has the same effect on the chords as discussed with the intervals.

The fundamental major chord is the most natural of all chords, since the components may be obtained within the first five partials of the root by lowering the fifth and third order partials by two and one octaves. For example (see Figure 4, Page 13), E₁" and G' are upper partials of C. In the fundamental minor chord, the sixth order partial of the fundamental tone is also the fifth order partial of the minor third and the fourth order partial of the fifth. For example, G" is a lower upper partial for each of C, E¹_b and G. Every consonant chord will be either a major or minor chord and may be formed from the fundamental chord of the major or minor by transposing or adding the octaves above or below some or all of its three tones.

It is interesting to note that the chord C:E₁:A¹_b is considered dissonant because it contains the diminished fourth E₁ : A¹_b (428 cents). On equal tempered instruments, as the piano, this interval is E : Ab or E : G# (400 cents) and is a major third. Therefore, on the piano, the chord C:E:Ab might be consonant since each one of its tones forms a consonant interval with each one of the other two, yet this chord is one of the harshest dissonances. Thus, even with the imperfect tuning of the piano, the original meaning of the intervals asserts itself.

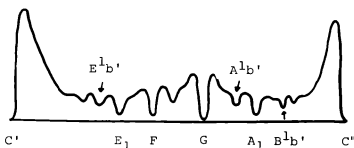
C (0)	fifth G 2/3 702	fourth F 3/4 498	major sixth A ₁ 3/5 884	major third E ₁ 4/5 386	minor third E ^{1b} 5/6 316
fourth F 3/4 498	major second 8/9 204				
major sixth A ₁ 3/5 884	major second 9/10 182	<u>major third</u> 3 4/5 386			
major third E ₁ 4/5 386	<u>minor third</u> 1 5/6 316	minor second 15/16 112	<u>fourth</u> 6 3/4 498		
minor third E ^{1b} 5/6 316	<u>major third</u> 2 4/5 386	major second 9/10 182	sharp fourth 18/25 568	minor second 24/25 70	
minor sixth A ^{1b} 5/8 814	minor second 15/16 112	<u>minor third</u> 4 5/6 316	minor second 24/25 70	diminished fourth 25/32 428	<u>fourth</u> 5 3/4 498

FIGURE 5

Chords contained within an octave. This chord consists of the fundamental tone C, one tone from the first horizontal line, and one tone from the first vertical column. Where the line and column intersect are the relations of these two tones. The name is underlined when the interval is consonant otherwise it is dissonant.

SUMMARY OF PART I

We started from the acoustical phenomena of upper partial tones, combinational tones and beats. We discussed methods for observing upper partial tones, and we found that for the better musical qualities of tone, the higher partial tones from the seventh onward must be weak. We found that consonance is determined by the ratios of small whole numbers, and that the physical effect of these ratios is sensible to the ear by the continuous or intermittent sensation of the auditory nerves. Rapid beats feel rough and unpleasant, because every intermittent excitement of our nervous system affects us more powerfully than one that lasts unaltered. Shown below is the degree of dissonance (based on a graph by Helmholtz) of two violin tones sounding together. The lower tone C' sounds continuously, while the other, starting from unison, glides slowly up through one octave to C'' . The distance of the curve from the bottom line indicates the roughness of the corresponding interval. This theory of consonance is a systematization of observed facts.



PART II

Up to this point the discussion has been purely physical. We have dealt solely with natural phenomena where nothing was arbitrary. Now we pass on to the domain of aesthetics and pay attention to artistic contrasts and means of expression. Helmholtz says, "The system of scales, modes and harmonic tissues does not rest solely upon inalterable natural laws, but is also the result of esthetical principles, which have already changed, and will still change with the progressive development of humanity."

SCALES

Having discussed musical tones of single strings, the harmoniousness of intervals and chords, we now discuss modes and scales. By taking an ascending series of 12 perfect fifths (702 cents) from F to E#, we get

F C G D A E B F# C# G# D# A# E#.

The E# is a little sharper (24 cents) than F seven octaves higher than the initial tone, [$12(702) - 7(1200) = 24$]. The difference of 24 cents is called the Pythagorean c and we will consider three ways to handle it.

CASE I. This difference is quite insensible and when completely neglected gives a scale based on perfect fifths and octaves. Once the frequency of a single note in the series of fifths is fixed, all the notes in both directions become fixed, giving the diatonic (8 notes per octave) scale.

(cents)

C (204)	D (204)	E (90)	F (204)	G (204)	A (204)	B (90)	C'
1	8/9	64/81	3/4	2/3	16/27	128/234	1/2

This is the Pythagorean intonation developed by Pythagoreas and the ancient Greeks. The major thirds (408 cents; 64/81) are all 22 cents (a Didymus comma) sharper than the natural major thirds (386 cents; 4/5), and the minor thirds (294 cents; 27/32) are 22 cents flatter than the natural minor thirds (316 cents; 5/6). Therefore, the Pythagorean scale is best suited to melody only as the harmony (of thirds and sixths) could be better from another choice of intonation. This intonation is easily tuned on the hammered dulcimer when the treble bridge is placed at the perfect fifth interval; one has merely to tune intervals of an octave (see Supplement, section on Tuning).

If we continue the series of fifths to the left from F, we have

. . . Ab Eb Bb F.

Now, in this intonation, there is a distinct difference of

meaning between sharps and flats. A# is not the same note as Bb as they differ by the Pythagorean comma of 24 cents. This intonation is therefore called linear as the series of fifths does not repeat itself. For Pythagorean intonation (and Meantone intonation), our present day musical staff notation was invented.

CASE II. The next scale to be considered is the natural or just scale which was developed in the 16th century. This scale is arrived at by seeking a scale rich in all consonances with the tonic of the scale. By taking an ascending series of four fifths from F, $[4(702) - 2(1200) = 386 + 22]$,

F C G D A,

we come to the tone A, which is $81/80$ (22 cents) or a Didymus comma higher than the natural major third of F, which is represented by A_1 . The inferior number of 1, 2, 3, etc. attached to the letter will represent a tone which is 1, 2, 3, etc. commas lower in pitch than the same letter without the inferior number attached (in Pythagorean intonation). Every superior number shall raise its tone by the comma. This notation has been used throughout.

From the Pythagorean series of fifths, we then proceed to the natural intonation of thirds by interrupting the series of perfect fifths with an imperfect fifth as in

F (702) C (702) G (702) D (680) A_1 (702) E_1 (702) B_1

Rearranging, we get

(cents)									
C (204)	D (182)	E_1 (112)	F (204)	G (182)	A_1 (204)	B_1 (112)	C'		
1	$8/9$	$4/5$	$3/4$	$2/3$	$3/5$	$8/15$	$1/2$		

Now this is a scale containing more of the consonant intervals in their simplest ratios. This is the best scale devised so far for pure harmonies.

By extending this series of fifths to the right from B_1 , we have

$F\#_1$ $C\#_1$ $G\#_1$ $D\#_1$ $A\#_1$ $E\#_1$

Now $E\#_1 = F + 2$ cents. Therefore, this intonation is also linear. For instruments with a fixed number of tones, the natural scale is burdened by requiring too many notes per octave (117, Ellis) for modulation into all keys.

CASE III. For this case, we take the difference of 24 cents and divide it equally among all the fifths. The linear series of 12 perfect fifths therefore becomes a circle of 12 'equal fifths', all 2 cents flat or 700 cents, $[12(700) - 7(1200) = 0]$.

		C	
E#	= F		G
	A#		D
D#			A
	G#		E
	C#		B
		F#	

This gives us the chromatic equal tempered scale played on most all instruments today. The equal tempered diatonic scale is

C (200) D (200) E (100) F (200) G (200) A (200) B (100) C'

By continuing this circle of fifths counterclockwise from F, we find that E# = F, Bb = A#, Eb = D#, and so on. Thus, this scale is cyclic, and the distinction between sharps and flats (A# and Bb) is no longer important. Now, with just 12 notes per octave, we can modulate into all keys, but we have sacrificed the pureness of the harmony of natural intonation. The pure consonances of the Pythagorean system are not injured to any extent worth notice by equal temperament. The inferior and superior numbers added to the letters representing the notes do not apply to equal temperament.

MODES

The modes to be considered are as follows:

(1) Major mode

C	D	E ₁	F	G	A ₁	B ₁	C'
1	8/9	4/5	3/4	2/3	3/5	8/15	1/2

(2) Mode of minor third (minor)

C	D	E ^{1b}	F	G	A ^{1b}	B ₁	C'
1	8/9	5/6	3/4	2/3	5/8	8/15	1/2

(3) Mode of the fourth

C	D	E ₁	F	G	A ₁	B ^{1b}	C'
1	8/9	4/5	3/4	2/3	3/5	5/9	1/2

(4) Mode of minor seventh

C	D	E ^{1b}	F	G	A ₁	B ^{1b}	C'
1	8/9	5/6	3/4	2/3	3/5	5/9	1/2

(5) Mode of minor sixth

C	D ^{1b}	E ^{1b}	F	G	A ^{1b}	Bb	C'
1	15/16	5/6	3/4	3/2	5/8	9/16	1/2

These modes can be understood by a consistent method of derivation if we consider the following scales composed of the consonant intervals of major third, fourth, fifth and major sixth:

<u>Ascending</u> <u>Related to:</u>		<u>Major</u> <u>Third</u>	<u>Fourth</u>	<u>Fifth</u>	<u>Major</u> <u>Sixth</u>
Tonic	C	E ₁	F	G	A ₁
Dominant	G	B ₁	C	D	E ₁
Subdominant	F	A ₁	Bb	C	D ₁

Descending (inverse of ascending)

Tonic	C'	A ¹ b	G	F	E ¹ b
Dominant	G'	E ¹ b	D	C	B ¹ b
Subdominant	F'	D ¹ b	C	Bb	A ¹ b

The modes are thus derived by combining these scales as follows:

- (1) Ascending C and G
- (2) Descending C and G (with B₁ from ascending G)
- (3) Ascending C and F
- (4) Ascending C and F (with E¹b from descending C)
- (5) Descending C and F

The tone B¹b, the minor seventh, is often replaced by B₁, the major seventh. Harmonic considerations favor B₁ since it is a partial of the compound tone of G, the dominant. Thus, some part of the consistency of the scales is sacrificed in order to bind the harmony closer together. In melodic progressions, the major seventh is favored in ascending passages going to the tonic. The major seventh is also called the leading tone and is important for preparing for or pushing on to the tonic. The minor seventh cannot be used to do this. Thus, the interval most distantly related to the tonic, the major seventh, has an important function in the scale.

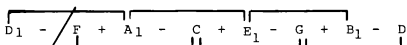
As long as one part music or melody alone is considered, all these modes are equally justified. However, the major mode has the closest and simplest relationship of the tones of all these modes.

HARMONY OF THE MODES

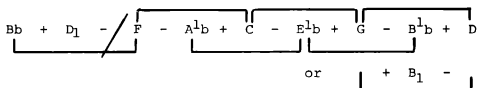
Since all consonant three note chords, when reduced to their closest position and simplest form, consist of a major and minor third, all the consonant chords of any key can be found by simply arranging them in order of thirds, as in the following tables. The braces above point out minor chords; the braces below indicate major chords. The + sign

represents a major third and the - sign, a minor third.

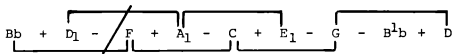
(1) major



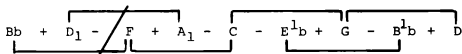
(2) minor



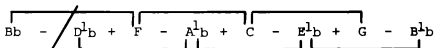
(3) Mode of fourth



(4) Mode of minor seventh



(5) Mode of minor sixth



The different intonations of the second and the seventh (D_1 , the grave second, and Bb) of the key are added to the left of the slash (/), since they can be used to imply modulation into other modes when the natural scale is used. These tones form chords which are not directly related to the tonic chord.

For the major and minor modes, there is a close connection among all the chords in a mode similar to the close connection among the tones of the scale. That is, the consonant chords are related to the tonic chord in a manner analogous to that which the notes of the scale are related to the tonic note. The other modes do not properly satisfy such conditions. The minor mode is actually less consonant than the major mode for natural intonation, but not for equal temperament.

The minor scale contains an interval $A^{1b} : B_1$ with numerical ratio 75 : 64 (274 cents) which is very strange. To make the scale more melodic, it sometimes has a different form in descending (given above) than it has in ascending

(given below).

C D E^{1b} F G A₁ B₁ C'

The mode of the fourth or mixolydian is principally distinguished from the major mode by its minor seventh (B^{1b} in the key of C). The leading tone, B₁, may be used in ascending passages provided B^{1b} is used often enough in descending passages. The effect of a mode is destroyed when an essential tone is changed at the close of a piece.

The mode of the minor seventh is distinguished from the minor mode by having a major chord on its subdominant F (F+A₁-C). The ascending minor scale belongs to the mode of the seventh, in which the leading note, B₁, is used.

The mode of the minor sixth is truly unique since it oscillates between major and minor. The minor second acts as a leading note to the tonic for descending progressions. Indeed, the whole scale is the inverse of the major scale. There is no dominant chord within this mode although G+B₁-D can be used.

DISCORDS

Discords are the opposite of concords, or chords with dissonant intervals. We have seen so far that consonance is the backbone of the construction of our musical scales. Consonances thus have an independent right to exist. They cause an agreeable kind of gentle and uniform excitement to the ear, whereas dissonances are distressing and cause intermittent sensations to the ear. Dissonances do, however, have a right to exist in music because they bring the beauty of consonance into relief by contrast and help to gain a more powerful means of expression. When the ear has been distressed by dissonances, it longs to return to the calm of consonances. In this way, dissonances are allowable as transitions between consonances.

For example, chords of the seventh contain dissonant intervals. The dominant seventh consists of the dominant, its third, fifth and seventh. Arranging the tones of a scale as a series of thirds, we see that a fifth is composed of two and a seventh three thirds.

chord of the seventh

F + A₁ - C + E₁ - G + B₁ - D ... F

dominant seventh

The sign ... represents the Pythagorean minor third (294 cents). The interval G to F is a minor seventh and is considered dissonant. Chords of the seventh are very popular in modern music and are one of the least dissonant discords.

KEYS AND MODULATION

The tone in the scale chosen to be the tonic (or the key) must bring the compass of the tones of the piece of music to be played within the compass of the singers or instruments. The key is simply the tonic note.

In many pieces of music, it is important to be able to make a temporary change of tonic, that is, to modulate into other keys. Just as consonances are made more prominent and effective by means of dissonances, the feeling for the tonic and the satisfaction which arises from hearing it can be heightened by temporary transitions to different keys. This adds great variety to musical expression.

COMPARISON OF INTONATIONS

The principal fault with equal tempered intonation is not the imperfect fifths, but in the thirds, as any deviation from the natural major third ($4/5$) is noticed in chords. Equally tempered or Pythagorean chords sound beside natural chords rough, dull, trembling or restless. The difference is so marked that everyone, whether musically cultivated or not, observes it at once. Chords of the dominant seventh in natural intonation have nearly the same degree of roughness as a common major chord of the same frequency in equal tempered intonation. In Pythagorean intonation, the character of the chord of the dominant seventh is brought out more distinctly by the sharpness of the leading tone. This distinct character of the leading tone is also felt in melodic music.

Modern musicians, with rare exceptions, have only heard music executed in equal temperament and they make light of its inexactness. The errors of the fifths are very small. The errors of the thirds are more important because they are much less perfect consonances, but only for melody. In consonant chords, every tone is equally sensitive to false intonation and the bad effects of equal tempered chords depends especially on the imperfect thirds.

Helmholtz says that the singer who practices to a tempered instrument has no principle at all for exactly and certainly determining the pitch of their voice. Beats can be used when practicing to an instrument tuned in natural intonation. Ellis says that it is difficult for any ear brought up among equal tempered thirds and sixths (which are dissonant for sustained tones) to understand the real distinction between consonance and dissonance. In natural intonation, modulations become much more expressive and the intensity of dissonances is much increased by their contrast with perfect chords.

Modern violin players aim at producing equal tempered intonations although this cannot be completely attained, owing to the perfect fifths of the open strings. The sole exception is when they play double-stops. The notes have to be somewhat differently stopped from what they played alone. The reason is the player is responsible for the harmoniousness of the interval in double-stops and this can

TABLE OF NATURAL INTONATION *

interval	note	ratio	cents	frequency
unison	C	1/1	0	264.0
minor seconds	D ¹ _b	15/16	112	281.6
	C ₂ [#]	24/25	70	275.0
major seconds	D ₁	9/10	182	293.3
	D ¹	8/9	204	297.0
minor thirds	D ₂ [#]	64/75	274	309.4
	E _{1b}	5/6	316	316.8
major thirds	E ₁	4/5	386	330.0
	E ¹	64/81	408	334.1
fourths	F	3/4	498	352.0
sharp fourths	F ₂ [#]	18/25	568	366.7
	F ₁ [#]	32/45	590	371.3
	G _{1b}	45/64	610	375.5
fifths	G ₁	27/40	680	391.1
	G ¹	2/3	702	396.0
minor sixths	G ₂ [#]	16/25	772	412.5
	A _b	81/128	792	417.2
	A _{1b}	5/8	814	422.4
major sixths	A ₁	3/5	884	440.0
	A ¹	16/27	906	445.5
minor sevenths	B _b	9/16	996	469.3
	B ¹ _b	5/9	1018	475.2
major sevenths	B ₁	8/15	1088	495.0
	B ¹	128/243	1110	501.2
octave	C'	1/2	1200	528.0

*This and the following table are for the precise comparison of all the intervals discussed in this book.

be verified by any violin player by the following fact: Tune the strings of the violin in perfect fifths, G D A E, and find where the finger must be placed on the A string to produce the B which will give a perfect fourth, B:E. Now, without moving the finger, strike this same B together with the open D string. The interval, D:B, would, according to the usual view, be a major sixth, but it would be a Pythagorean one (906 cents). In order to obtain the more consonant sixth, D:B₁ (884 cents), the finger would have to be drawn back about 3/20 inch (0.38 cm). This is an

TABLE OF PYTHAGOREAN INTONATION AND EQUAL TEMPERAMENT

note	Pythagorean intonation			equal temperament		
	ratio	cents	frequency	ratio	cents	frequency
C	1/1	0	260.7	1/1	0	261.6
Db	234/256	90	274.6	1/ $\sqrt[12]{2}$	100	277.2
C#	211/37	114	278.4	"	"	"
D	8/9	204	293.3	1/ $\sqrt[12]{4}$	200	293.7
Eb	27/32	294	309.0	1/ $\sqrt[12]{8}$	300	311.1
E	64/81	408	330.0	1/ $\sqrt[12]{16}$	400	329.6
F	3/4	498	347.6	1/ $\sqrt[12]{32}$	500	349.2
F#	512/729	612	371.2	1/ $\sqrt[12]{64}$	600	370.0
G	2/3	702	391.2	1/ $\sqrt[12]{128}$	700	392.0
Ab	81/128	792	411.9	1/ $\sqrt[12]{256}$	800	415.3
G#	212/38	816	417.7	"	"	"
A	16/27	906	440.0	1/ $\sqrt[12]{512}$	900	440.0
Bb	9/16	996	463.4	1/ $\sqrt[12]{1024}$	1000	466.2
B	128/243	1110	495.0	1/ $\sqrt[12]{2048}$	1100	493.9
C'	1/2	1200	521.4	1/2	1200	523.2

appreciable distance and is sufficient to alter the beauty of the consonance most preceptibly.

Hemholtz says that the intervals of natural intonation are really natural for uncorrupted ears. Moreover, the deviations of equal temperament are really perceptible and unpleasant to uncorrupted ears when compared to natural intonation and that singing by natural intervals is much easier than singing in equal temperament. The complicated system of intervals and the large number of notes per octave (for modulation) which the natural scale necessitates, and which undoubtedly much increases the manual difficulty of performance on instruments with fixed tones, does not exist for either singer or violinist. They let themselves be guided by their ear and proceed by the intervals of the natural diatonic scale. Only the theoretician finds the calculations complicated.

As long as it is necessary practically to limit the number of separate tones within the octave to 12, there can be no question at all as to the superiority of equal temperament with its 12 equal semitones, and, as a natural consequence, it is the acknowledged method of tuning. The bowed instruments, with their perfect fifths, still deviate

from it. Equal tempered intonation was first and especially developed on the pianoforte and hence gradually transferred to other instruments. Emanuel Bach said that a correctly tuned piano has the most perfect intonation of all instruments. Almost all instrumental music is designed for rapid movement and this is its essential advantage over vocal music. Furthermore, in rapid passages, the evils of equal temperament are but little apparent. In fact, maybe instrumental music has been forced into rapidity of movement by equal tempered intonation because the full harmoniousness of slow chords of natural intonation from singers was not possible.

AESTHETICAL RELATIONS

The aesthetics of music is concerned with melody, harmony, rhythm, forms of composition of pieces and means of musical expression. The essential basis of music is melody. Harmony has become, during the last four centuries, an essential and indispensable means of strengthening melodic relations, but finely developed music existed for thousands of years (and still exists) without any harmony at all. Harmony has become so important to modern music, however, that some musicians assert, "Melody is resolved harmony." Actually, both melody and harmony have a common cause in the natural formation of musical tones (i.e. partial tones).

A feeling for the melodic relationship of consecutive tones was first developed starting with the octave and fifth (Pythagorean scale) and advancing to the third (natural scale). This feeling of relationship was founded on the perception of identical partial tones in the corresponding compound tones. The octave and fifth are obvious to beginners, with the thirds requiring more practice.

After musicians had long been content with the melodic relationship of tones, they began in the middle ages to make use of harmonic relationship. The effect of various combinations of tones depends partly on the identity or difference of two of their different partial tones and partly on combinational tones. Now, in melodic relationship, the equality of the upper partial tones can only be perceived by remembering the preceding compound tone, but, in harmonic relationship, it is determined by immediate sensation, the presence or absence of beats. Thus, there is greater liveliness in harmonic combinations of tone than in melodic and there is a greater wealth of clearly perceptible relations of tones through chords and discords.

Beats are easy to recognise as such when they occur slowly, but those which characterize dissonances are, almost without exception, very rapid. Even then the beats are partly covered by partials which do not beat. Therefore, as a general rule, the hearer is perfectly unconscious of the cause to which the roughness of dissonance is due. On the hammered dulcimer, the note is powerful only at the moment when it is struck and rapidly decreases in loudness, so that the beats which characterize the dissonances do not have

time to become sensible in the beginning, but only after the tone is greatly diminished in intensity. This allows for the use of dissonant intervals which would be perfectly insupportable on other instruments.

Instruments which can vary the force or the intensity of tone at will, as the violin or voice, produce a uniform or varying sensation in the ear and give the most expressive performance. On the other hand, instruments which are struck, as the hammered dulcimer, have a peculiar value for clearly defining the rhythm.

The development of harmony gives rise to a much richer form of musical art than is possible with melody. Also, it allows for modulation into more distant keys. There is a feeling for the relationship of chords to one another and to the tonic chord which is analogous to the relationship of notes to the tonic note. As the relationship of compound tones depends on the identity of two or more partial tones, so do the relationship of chords depend on the identity of two or more notes. For the musician, this relationship of chords (and keys) is much more intelligible than that of compound tones since the musician readily hears the identical tones or sees them in musical notation. But, the uninstructed hearer is unconscious of the reason of the connection of a clear and agreeable series of chords or a well-connected melody. This hearer is startled by a false series of chords or notes and feels its unexpectedness, but is not necessarily conscious of the reason.

An aesthetical principle the ancient world developed for melody, and the modern world for harmony, is that the whole mass of musical tone must stand in a close and distinctly perceptible relationship to the tonic, must be developed from this tonic and must finally return to it. Concerning this, the ear is more satisfied with a closing major chord the more closely the order of the tones used imitates the arrangement of the partial tones of the tonic (see Figure 1, Page 2).

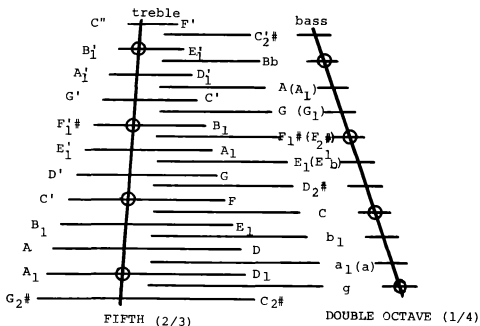
The resemblance of an octave to its root note is so great and striking that the duller ear perceives it; the octave seems to be almost a pure repetition of partials of the root as it merely repeats the even ordered partials of the root, without adding anything new. Hence, the aesthetical effect of an octave is very simple, but little attractive. The most attractive of the intervals (melodically and harmonically) are clearly the thirds and sixths; these are the intervals which lie at the very boundary of those that the ear can grasp. The major third and major sixth can best be appreciated when the first six partial tones are audible. When these partial tones have little force, the sharp thirds of equal temperament and Pythagorean intonation are not so noticeable.

APPENDIX I

Natural Intonation for the Hammered Dulcimer

This tuning for the dulcimer can be used to hear many of the facts discussed in this book. This scheme furnishes power to play in G and C major, & F major without the grave second G_1 . Also in E_1 major without the grave second $F_2^\#$. It can modulate perfectly from C major to the dominant G major, and from G major to its relative minor E_1 . It could modulate perfectly from C major to its relative minor A_1 if $F_2^\#$ were tuned for the subdominant. It can modulate perfectly from G major to its subdominant C major, and thence to its subdominant C major, and thence to its subdominant F major, if G_1 is tuned. Thus, a considerable variety of harmony and modulation is open to this scheme, but, of course, each piece requires special arrangement.

To tune, make the 8 major chords CE_1G , GB_1D , $DF_1^\#A$, FA_1C , BbD_1F , $A_1C_2^\#E_1$, $E_1G_2^\#B_1$, and $B_1D_2^\#F_1^\#$ perfect without beats. A_1 is equal to 440 cycles/sec. Some of these notes may be changed to give a wider range of octaves to some keys (as with F and $F_1^\#$).



APPENDIX II

Aeolian Dulcimer

Design your own musical sound box to fit in a window in such a manner that the wind will blow directly across 4 or 5 strings (each a different diameter, tuned to the same note or its octaves). The instrument (Aeolian dulcimer) will sound different partial tones for different wind velocities. When the air moves with velocity V around a long narrow obstacle of diameter D , f whirlwinds per second are generated in the air where

$$f = 5 V / 27 D$$

These whirlwinds are heard from the rigging of boats or elevated wires when the wind blows hard. If a natural frequency of the obstacle is equal to the frequency of the whirlwinds, then the obstacle vibrates with this frequency, and the sound can be often heard even when the wind blows softly. The natural frequencies of a wire tuned to C are given by the frequencies of the partial tones.

APPENDIX III

Cents

Musical intervals depend on frequency ratios. Therefore, in order to add or subtract musical intervals, we must multiply or divide the corresponding ratios. The 'cent' is a unit in which all musical intervals can be measured and the measurements added or subtracted in the ordinary way. We thus define

$$\text{musical interval} = 1200 \log_2 (f_1/f_2) \text{ cents}$$

where f_1 and f_2 are the frequencies of the two notes of the interval ($f_1/f_2 = 2$ for the octave). This scale has 100 cents to the tempered semitone, 200 cents to the tempered tone and 1200 cents to the octave. It is a convenient size of unit since 1 cent represents approximately the smallest interval that the very best ear can appreciate. This scale is the most widely used in the published literature on intervals, and also on instruments designed to measure musical intonation. Slight discrepancies in the cents are due to roundoff to the nearest cent.

INDEX

- Absorption of sound, 7, 8
- Acoustics of sound, 7, 8
- Aeolian dulcimer, 29
- Amplitude, 1, 2
- Ascending scale, 20
- Asymmetrical distortion, 9

- Bach, E., 26
- Beats, 9-12, 16, 26

- Cents, 29
- Chords, 14-16
- Chromatic scale, 19
- Combinational tones, 9-11, 26
- Comma of Didymus, 17
- Comma of Pythagoras, 17
- Compound tone, 1, 11
- Consonance, 11-16, 22
- Constructive interference, 10
- Cyclic scale, 19

- Defect, 12
- Descending scale, 20
- Destructive interference, 10
- Didymus comma, 17
- Difference tones, 9
- Discord, 22
- Dissonance, 7, 11-16, 26
- Dominant, 3, 20
- Dominant seventh, 22, 23

- Ear, 9, 22
- Ellis, A. J., 18, 23
- Equal temperament, 18, 19, 23, 25, 27

- Flageolets, 9
- Force of tone, 1
- Frequency, 1
- Fundamental, 14

- Grave second, 21
- Greek, 17

- Hammered dulcimer, 4-6, 9, 11, 26-28
- Harmonics, 3, 9
- Harmony, 20-22, 25-27
- Helmholtz, H., 11, 16, 17, 23, 25, 26

- Inharmonic partials, 3, 9
- Interference of sound, 10

- Just, see Natural

- Key, 23, 27

- Linear scales, 18
- Leading tone, 20, 22, 23

- Major chord, 1-3, 27
- Major scale, 3, 19-21
- Minor chord, 14
- Minor scale, 19, 21, 22
- Modes, 17, 19-22
- Modulation, 23, 25, 19
- Musical tone, 1
- Myxolydian, 22

- Natural intonation, 3, 18, 19, 24, 28
- Natural harmonics, 3
- Nodes of vibration, 4

- Ohm's law, 3

- Partial tone, 1
- Pianoforte, 6, 7, 14, 26
- Pitch, 1
- Prime tone, 1
- Proper tone, 6
- Pythagorean comma, 17
- Pythagorean intonation, 17, 19, 23, 25, 27

- Quality of tone, 1, 6-7

- Resonance, 4, 6, 29
- Reverberation time, 7, 8
- Rhythm, 26, 27
- Root, 12, 27

- Scales, 17
- Simple tone, 1, 6
- String, plucked, 3
 struck, 3, 4
- Subdominant, 3, 20
- Summation tone, 9
- Sympathetic resonance, 4

- Tonic, 3, 27
- Transverse vibration, 2
- Tuning fork, 6, 10

- Violin, 6, 7, 9, 16, 25, 27

- Whirlwinds in air, 29

GLOSSARY

Aeolian-caused by the winds.

Chromatic scale-twelve tones to an octave.

Coefficient-a number or letter put before an expression and multiplying it.

Diatonic scale-eight tones to an octave.

Diminished interval-an interval that is shorter than the interval indicated.

Dominant-fifth tone of a diatonic scale.

Grave-lower in pitch.

Interval-the difference in pitch between two tones; also the effect of two tones heard at the same time.

Major key-mode with major third.

Minor key-mode with minor third.

Mode- method of dividing an octave into arbitrary steps and half-steps.

Modulation-a change of pitch.

Node-a stationary point in a vibrating string.

Scale-tones of a key in regular order.

Semitone-half step

Subdominant-the fourth tone of a diatonic scale.

Tonic-key note or first note in a scale.

ERRATA: Preface-*The Hammered Dulcimer* should be *Rick Fogel Hammered Dulcimers*.

Page 24-G₁b should be G¹b.

"As far as the laws of mathematics
refer to reality, they are not certain;
and as far as they are certain they do
not refer to reality." A. Einstein

Published By



1916 Pike Place #906
Seattle, WA 98101
(206) 910-8259